

# Finding Zeros of Polynomial Functions WS#1

①  $f(x) = 2x^4 - 7x^3 + 14x^2 + 40x; 2$   
 $= x(2x^3 - 7x^2 - 14x + 40)$

$$\begin{array}{r|rrrr} 2 & 2 & -7 & -14 & 40 \\ & & 4 & -6 & -40 \\ \hline & 2 & -3 & -20 & 0 \end{array}$$

$$2x^2 - 3x - 20 = 0$$

$$(2x + 5)(x - 4) = 0$$

$$2x + 5 = 0 \quad x - 4 = 0$$

$$2x = -5 \quad x = 4$$

$$x = -\frac{5}{2}$$

zeros:  $-\frac{5}{2}, 0, 2, 4$

②  $f(x) = 6x^4 - 31x^3 + 4x^2 + 5x; \frac{1}{2}$   
 $= x(6x^3 - 31x^2 + 4x + 5)$

$$\begin{array}{r|rrrr} \frac{1}{2} & 6 & -31 & 4 & 5 \\ & & 3 & -14 & -5 \\ \hline & 6 & -28 & -10 & 0 \end{array}$$

$$6x^2 - 28x - 10 = 0$$

$$3x^2 - 14x - 10 = 0$$

$$(3x + 1)(x - 5) = 0$$

$$3x + 1 = 0 \quad x - 5 = 0$$

$$3x = -1 \quad x = 5$$

$$x = -\frac{1}{3}$$

zeros:  $-\frac{1}{3}, 0, \frac{1}{2}, 5$

③  $f(x) = 5x^4 - 21x^3 - 21x^2 + 5x; 5$   
 $= x(5x^3 - 21x^2 - 21x + 5)$

$$\begin{array}{r|rrrr} 5 & 5 & -21 & -21 & 5 \\ & & 25 & 20 & -5 \\ \hline & 5 & 4 & -1 & 0 \end{array}$$

$$5x^2 + 4x - 1 = 0$$

$$(5x - 1)(x + 1) = 0$$

$$5x - 1 = 0 \quad x + 1 = 0$$

$$5x = 1 \quad x = -1$$

$$x = \frac{1}{5}$$

zeros:  $-1, 0, \frac{1}{5}, 5$

④  $f(x) = 15x^4 + 32x^3 + 3x^2 - 2x; -\frac{1}{3}$   
 $= x(15x^3 + 32x^2 + 3x - 2)$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 15 & 32 & 3 & -2 \\ & & -5 & -9 & 2 \\ \hline & 15 & 27 & -6 & 0 \end{array}$$

$$\div 3 \quad 5 \quad 9 \quad -2$$

$$5x^2 + 9x - 2 = 0$$

$$(5x - 1)(x + 2) = 0$$

$$5x - 1 = 0 \quad x + 2 = 0$$

$$5x = 1 \quad x = -2$$

$$x = \frac{1}{5}$$

zeros:  $-2, -\frac{1}{3}, 0, \frac{1}{5}$

# Finding Zeros, pg 2

⑤  $f(x) = 2x^4 - 15x^3 + 22x^2 + 15x + 5$   
 $= x(2x^3 - 15x^2 + 22x + 15)$

$$\begin{array}{r|rrrr} 5 & 2 & -15 & 22 & 15 \\ & & 10 & -25 & -15 \\ \hline & 2 & -5 & -3 & 0 \end{array}$$

$$2x^2 - 5x - 3 = 0$$

$$(2x + 1)(x - 3) = 0$$

$$2x + 1 = 0 \quad x - 3 = 0$$

$$2x = -1 \quad x = 3$$

$$x = -\frac{1}{2}$$

zeros:  $-\frac{1}{2}, 0, 3, 5$

⑥  $f(x) = 2x^4 - 15x^3 + 37x^2 - 30x + 2$   
 $= x(2x^3 - 15x^2 + 37x - 20)$

$$\begin{array}{r|rrrr} 2 & 2 & -15 & 37 & -20 \\ & & 4 & -22 & 20 \\ \hline & 2 & -11 & 15 & 0 \end{array}$$

$$2x^2 - 11x + 15 = 0$$

$$(2x - 5)(x - 3) = 0$$

$$2x - 5 = 0 \quad x - 3 = 0$$

$$2x = 5 \quad x = 3$$

$$x = \frac{5}{2}$$

zeros:  $0, 2, \frac{5}{2}, 3$

⑦  $f(x) = 9x^4 - 66x^3 + 115x^2 - 50x + \frac{5}{3}$   
 $= x(9x^3 - 66x^2 + 115x - 50)$

$$\begin{array}{r|rrrr} \frac{5}{3} & 9 & -66 & 115 & -50 \\ & & 15 & -85 & 50 \\ \hline & 9 & -51 & 30 & 0 \\ \div 3 & 3 & -17 & 10 & \end{array}$$

$$3x^2 - 17x + 10 = 0$$

$$(3x - 2)(x - 5) = 0$$

$$3x - 2 = 0 \quad x - 5 = 0$$

$$3x = 2 \quad x = 5$$

$$x = \frac{2}{3}$$

zeros:  $0, \frac{2}{3}, \frac{5}{3}, 5$

⑧  $f(x) = 25x^4 - 85x^3 - 58x^2 - 8x - \frac{2}{5}$   
 $= x(25x^3 - 85x^2 - 58x - 8)$

$$\begin{array}{r|rrrr} -\frac{2}{5} & 25 & -85 & -58 & -8 \\ & & -10 & 38 & 8 \\ \hline & 25 & -95 & -20 & 0 \\ \div 5 & 5 & -19 & -4 & \end{array}$$

$$5x^2 - 19x - 4 = 0$$

$$(5x + 1)(x - 4) = 0$$

$$5x + 1 = 0 \quad x - 4 = 0$$

$$5x = -1 \quad x = 4$$

$$x = -\frac{1}{5}$$

zeros:  $-\frac{2}{5}, -\frac{1}{5}, 0, 4$

# Finding Zeros, pg 3

⑨  $f(x) = 10x^4 - 41x^3 + 2x^2 + 8x; -\frac{2}{5}$   
 $= x(10x^3 - 41x^2 + 2x + 8)$

$$\begin{array}{r|rrrr} -\frac{2}{5} & 10 & -41 & 2 & 8 \\ & & -4 & 18 & -8 \\ \hline & 10 & -45 & 20 & 0 \\ \div 5 & 2 & -9 & 4 & \end{array}$$

$$\begin{aligned} 2x^2 - 9x + 4 &= 0 \\ (2x-1)(x-4) &= 0 \\ 2x-1=0 & \quad x-4=0 \\ 2x=1 & \quad x=4 \\ x &= \frac{1}{2} \end{aligned}$$

Zeros:  $-\frac{2}{5}, 0, \frac{1}{2}, 4$

⑩  $f(x) = 3x^4 - x^3 - 8x^2 - 4x; 2$   
 $= x(3x^3 - x^2 - 8x - 4)$

$$\begin{array}{r|rrrr} 2 & 3 & -1 & -8 & -4 \\ & & 6 & 10 & 4 \\ \hline & 3 & 5 & 2 & 0 \end{array}$$

$$\begin{aligned} 3x^2 + 5x + 2 &= 0 \\ (3x+2)(x+1) &= 0 \\ 3x+2=0 & \quad x+1=0 \\ 3x &= -2 & x &= -1 \\ x &= -\frac{2}{3} & & \end{aligned}$$

Zeros:  $-1, -\frac{2}{3}, 0, 2$

⑪  $f(x) = 15x^4 + 46x^3 + 36x^2 + 8x; \frac{2}{3}$   
 $= x(15x^3 + 46x^2 + 36x + 8)$

$$\begin{array}{r|rrrr} \frac{2}{3} & 15 & 46 & 36 & 8 \\ & & -10 & -24 & -8 \\ \hline & 15 & 36 & 12 & 0 \\ \div 3 & 5 & 12 & 4 & \end{array}$$

$$\begin{aligned} 5x^2 + 12x + 4 &= 0 \\ (5x+2)(x+2) &= 0 \\ 5x+2=0 & \quad x+2=0 \\ 5x &= -2 & x &= -2 \\ x &= -\frac{2}{5} & & \end{aligned}$$

Zeros:  $-2, -\frac{2}{3}, -\frac{2}{5}, 0$

⑫  $f(x) = 15x^4 - 44x^3 - 60x^2 - 16x; -\frac{2}{5}$   
 $= x(15x^3 - 44x^2 - 60x - 16)$

$$\begin{array}{r|rrrr} -\frac{2}{5} & 15 & -44 & -60 & -16 \\ & & -6 & 20 & 16 \\ \hline & 15 & -50 & -40 & 0 \\ \div 5 & 3 & -10 & -8 & \end{array}$$

$$\begin{aligned} 3x^2 - 10x - 8 &= 0 \\ (3x+2)(x-4) &= 0 \\ 3x+2=0 & \quad x-4=0 \\ 3x &= -2 & x &= 4 \\ x &= -\frac{2}{3} & & \end{aligned}$$

Zeros:  $-\frac{2}{3}, -\frac{2}{5}, 0, 4$

# Finding zeros, pg. 4

⑬  $f(x) = 2x^4 + x^3 - 26x^2 - 40x; -2$   
 $= x(2x^3 + x^2 - 26x - 40)$

$$\begin{array}{r|rrrr} -2 & 2 & 1 & -26 & -40 \\ & & 4 & 6 & 40 \\ \hline & 2 & -3 & -20 & 0 \end{array}$$

$$2x^2 - 3x - 20 = 0$$

$$(2x+5)(x-4) = 0$$

$$2x+5=0 \quad x-4=0$$

$$2x = -5 \quad x = 4$$

$$x = -\frac{5}{2}$$

zeros:  $-2, -\frac{5}{2}, 0, 4$

⑭  $f(x) = 3x^4 - 4x^3 - 75x^2 + 100x; 5$   
 $= x(3x^3 - 4x^2 - 75x + 100)$

$$\begin{array}{r|rrrr} 5 & 3 & -4 & -75 & 100 \\ & & 15 & 55 & -100 \\ \hline & 3 & 11 & -20 & 0 \end{array}$$

$$3x^2 + 11x - 20 = 0$$

$$(3x-4)(x+5) = 0$$

$$3x-4=0 \quad x+5=0$$

$$3x = 4 \quad x = -5$$

$$x = \frac{4}{3}$$

zeros:  $-5, 0, \frac{4}{3}, 5$

⑮  $f(x) = 3x^4 - 4x^3 - 35x^2 + 12x; -3$   
 $= x(3x^3 - 4x^2 - 35x + 12)$

$$\begin{array}{r|rrrr} -3 & 3 & -4 & -35 & 12 \\ & & -9 & 39 & -12 \\ \hline & 3 & -13 & 4 & 0 \end{array}$$

$$3x^2 - 13x + 4 = 0$$

$$(3x-1)(x-4) = 0$$

$$3x-1=0 \quad x-4=0$$

$$3x = 1 \quad x = 4$$

$$x = \frac{1}{3}$$

zeros:  $-3, 0, \frac{1}{3}, 4$

⑯  $f(x) = 5x^4 - 7x^3 - 81x^2 - 45x; -3$   
 $= x(5x^3 - 7x^2 - 81x - 45)$

$$\begin{array}{r|rrrr} -3 & 5 & -7 & -81 & -45 \\ & & -15 & 66 & 45 \\ \hline & 5 & -22 & -15 & 0 \end{array}$$

$$5x^2 - 22x - 15 = 0$$

$$(5x+3)(x-5) = 0$$

$$5x+3=0 \quad x-5=0$$

$$5x = -3 \quad x = 5$$

$$x = -\frac{3}{5}$$

zeros:  $-3, -\frac{3}{5}, 0, 5$